

# Binomial Expansion

In order to evaluate a binomial expression raised to the  $n^{\text{th}}$  power, one often distributes the binomial expression over itself  $n$  times. However as  $n$  becomes larger, this process becomes time-consuming, cumbersome, and mistakes in the distribution process typically become more frequent. The Binomial Expansion Theorem applies patterns to avoid distribution, thereby avoiding the time and frequent error involved with repetitive distribution.

General Pattern:  $(a + b)^n$

1. Write down the  $n^{\text{th}}$  row of Pascal's Triangle
2. Place the  $a$  term of the binomial expression after each number from Pascal's Triangle. The  $a$  that appears after the first number from Pascal's Triangle should be raised to the  $n^{\text{th}}$  power. The  $a$  that appears after the second number from Pascal's Triangle should be raised to the  $n-1^{\text{th}}$  power, and so on until the  $a$  that appears after the last number from Pascal's Triangle is raised to the 0 power.
3. Place the  $b$  term of the binomial expression after each  $a$  from step 2. The  $b$  that appears after the first number from Pascal's Triangle should be raised to the 0 power. The  $b$  that appears after the second number from Pascal's Triangle should be raised to the 1<sup>st</sup> power, and so on until the  $b$  that appears after the last number from Pascal's Triangle is raised to the  $n^{\text{th}}$  power.

Example 1:  $(x + 3)^5$

$$\begin{array}{l} 1. \quad 1 \qquad +5 \qquad +10 \qquad +10 \qquad +5 \qquad +1 \\ 2. \quad 1x^5 \quad +5x^4 \quad +10x^3 \quad +10x^2 \quad +5x^1 \quad +1x^0 \\ 3. \quad 1x^5 3^0 \quad +5x^4 3^1 \quad +10x^3 3^2 \quad +10x^2 3^3 \quad +5x^1 3^4 \quad +1x^0 3^5 \\ \quad 1x^5 \quad +15x^4 \quad +90x^3 \quad +270x^2 \quad +405x \quad +243 \end{array}$$

Example 2:  $(2x - 7)^3$

$$\begin{array}{l} 1. \quad 1 \qquad +3 \qquad +3 \qquad +1 \\ 2. \quad 1(2x)^3 \quad +3(2x)^2 \quad +3(2x)^1 \quad +1(2x)^0 \\ 3. \quad 1(2x)^3 (-7)^0 \quad +3(2x)^2 (-7)^1 \quad +3(2x)^1 (-7)^2 \quad +1(2x)^0 (-7)^3 \\ \quad 8x^3 \qquad -84x^2 \qquad +294x \qquad -343 \end{array}$$